Hybrid models for Active Noise Reduction

→ Physics-enhanced ML model architectures for nonlinear systems

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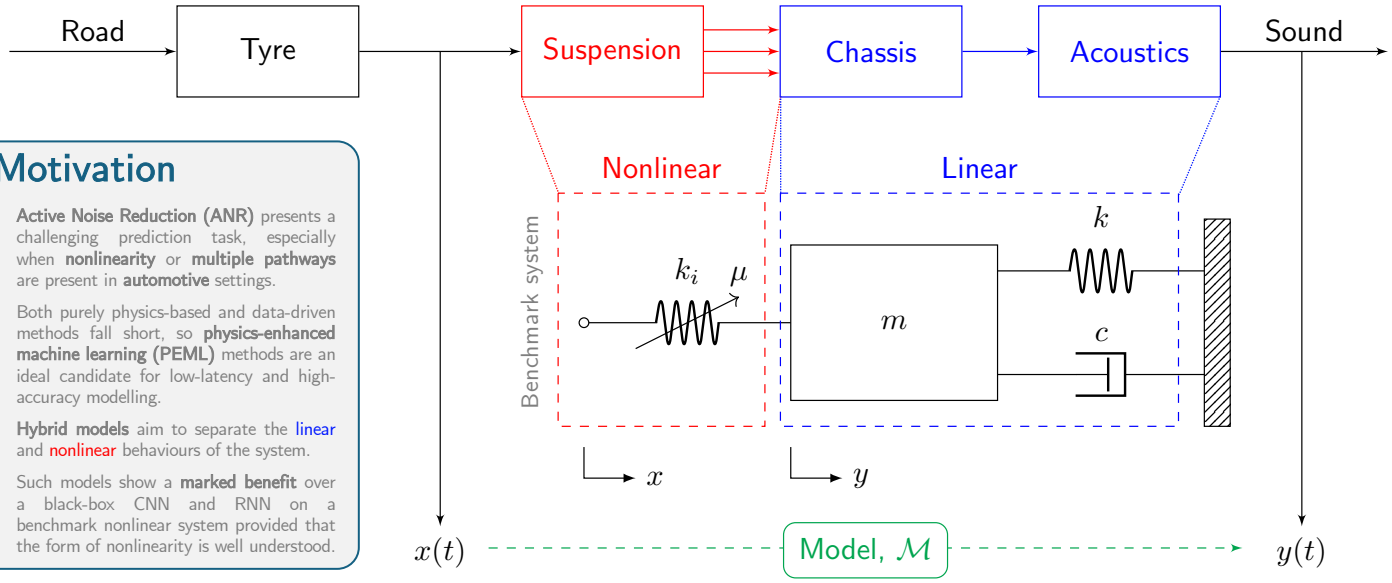


Motivation

- **Active Noise Reduction (ANR)** presents a challenging prediction task, especially when **nonlinearity** or **multiple pathways** are present in **automotive** settings.
- Both purely physics-based and data-driven methods fall short, so **physics-enhanced machine learning (PEML)** methods are an ideal candidate for low-latency and high-accuracy modelling.
- **Hybrid models** aim to separate the linear and nonlinear behaviours of the system.
- Such models show a **marked benefit** over a black-box CNN and RNN on a benchmark nonlinear system provided that the form of nonlinearity is well understood.

'Hybrid models'

- A *hybrid* model is one which aims to **separate system behaviours**, often to focus model complexity in a particular area of interest.
- For a mechanical system, separating the linear and nonlinear behaviour is desirable:
  - The linear model can leverage the known physics of a system (e.g. a known solution to the linearised governing ODE).
  - The nonlinear model can utilise data to learn the residual behaviour – the model type will depend on the prior knowledge of the system.
  - For an **abstracted automotive** system, the suspension is the nonlinear element, connected to the chassis and acoustics, which represent the linear part [1].
- Often these behaviours are coupled, and so careful architecture design is required.

Coherence

$$\gamma_{xy}^2 = \frac{|S_{xy}|^2}{S_{xx}S_{yy}}$$

Linear

- Where S_{xy} is the cross-spectral density between signals x and y.
- Describes 'how much of the output y can be described by a **linear model**'.

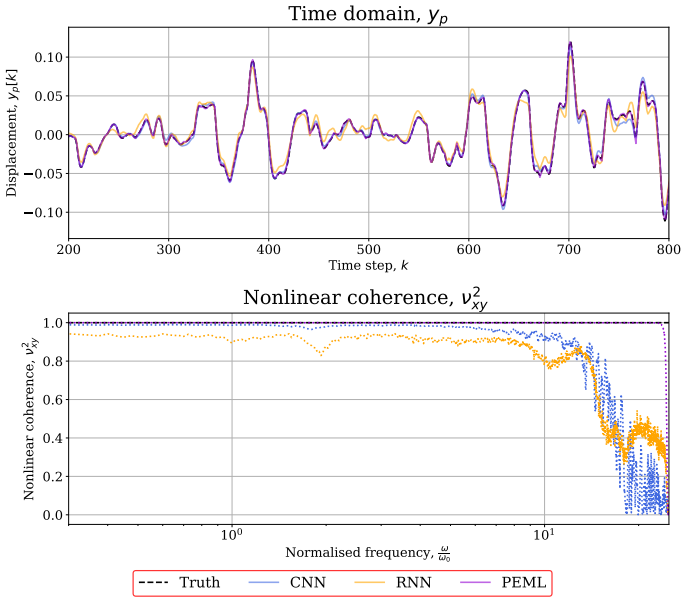
$$\nu_{xy}^2 = \gamma_{y_t y_p}^2 = \frac{|S_{y_t y_p}|^2}{S_{y_t y_t} S_{y_p y_p}}$$

Nonlinear

- Equal to the linear coherence between the truth y_t and model prediction y_p [2].
- Describes 'how much of the **nonlinearity is captured** by the model'.

Performance

- Three models: two black-box (**CNN**, **RNN**), one **PEML,** were trained on simulated time-series data (N=2000) [a] of the benchmark system with *moderate nonlinearity*.



Conclusions

1. Hybrid models can be used to **effectively separate** linear and nonlinear behaviours in an abstracted benchmark system.
2. Using this physics-enhanced approach can **improve model accuracy** compared to purely data-driven methods. This is evident in the **superior nonlinear coherence** across the frequency range, and in the normalised **test loss** in the time domain:

	CNN	RNN	PEML
L_{NMSE}	0.0139	0.0634	0.00138

- ... provided that the form of the nonlinearity is well understood.

Further work

- Modelling **more complex and challenging systems**, that have multiple pathways, time-dependent nonlinearities, and/or many more degrees of freedom.
  - This might use a range of specialised ML architectures for modelling the nonlinearity, especially if the form is uncertain or measurements are noisy.
- Validating use in practice for real-time control with an **experimental rig.** This should be closer to the car model itself, whilst still remaining tractable.

References

[1] M. De Brett, T. Butlin, L. Andrade, and O. M. Nielsen, 'Experimental investigation into the role of nonlinear suspension behaviour in limiting feedforward road noise cancellation', Journal of Sound and Vibration, Jan. 2022.

[2] J. Massingham, O. Nielsen, and T. Butlin, 'A method for identifying causality in the response of nonlinear dynamical systems', Sept. 26, 2024, arXiv: arXiv:2409.17872 [Online].

[a] Time series data was simulated using a Newmark-Beta scheme, generating N samples of length L in response to pulsed bandlimited white noise, providing N independent 'frames' of data separated for model training/validation.